

Canard-induced mixed mode oscillations in an excitable glow discharge plasmas

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We demonstrated experimentally canard induced mixed mode oscillations (MMO) in an excitable glow discharge plasma, and the results are validated through numerical solution of the FitzHugh Nagumo (FHN) model. When glow discharge plasma is perturbed by applying a magnetic field, it shows mixed mode oscillatory activity, i.e., quasiperiodic small oscillations interposed with large bounded limit cycles oscillations. The initial quasiperiodic oscillations were observed to change into large amplitude limit cycle oscillations with magnetic field, and the number of these oscillation increases with increase in the magnetic field. Fourier analysis of both numerical and experimental results show that the origin of these oscillations are canard-induced phenomena, which occurs near the threshold of the control parameter. Further, the phase space plots also confirm that the oscillations are basically canard-induced MMOs.

I. INTRODUCTION

It is well known that the change in a control parameters or external perturbation in a threshold or an excitable systems produces various nonlinear phenomena such as noise-induced resonances, canard oscillations and mixed mode oscillation, which have been observed experimentally as well as numerically in many physical, chemical, biological and electronics systems [1–17]. However, these kind of phenomena have been observed in very a few experiments in case of plasmas [18–22]. This is mainly because, it is not easy to achieve excitability condition in a plasma system. So far most of the nonlinear dynamical experiments, which depends on the excitability of plasma were performed mostly in the glow discharge plasma, and in these experiments the excitability has been achieved through Hopf bifurcation [18, 19] or homoclinic bifurcation [22]. Discharge voltage or discharge current was the control parameter (CP) for the above mentioned plasma experiments. When plasmas is perturbed externally at its excitable state by using noise or a periodic signal or together, the system shows stochastic resonance, frequency entertainments, period pulling and other perturbation-enhanced nonlinear phenomena [18–23], and one important feature of these experiments was that the coarse change in the CP was sufficient to generate these kind of phenomena. It is observed that if the change in the CP near the threshold is very small, then such excitable system may also generate canard-enhanced phenomena. For example, the FitzHugh Nagumo (FHN) model or real experiments generates canard, and various canard-enhanced phenomena due to small change in CP near the threshold that have already been studied in detail [1, 5, 16, 24–30].

The canard phenomena in an excitable system means

the generation of small quasiperiodic oscillations that has been observed through numerical simulation as well as in a few experiments for a small change in the CP near the threshold of excitability [1, 5, 14, 15]. An important feature of a system, which shows canard-induced phenomena is that minute change in the CP produces large bounded limit cycles of different frequency from small quasiperiodic oscillations [5]. Though minute change in the CP is easily attainable for the numerical experiments to get canard explosion, it is difficult to achieve such small change in the CP for a real experiment. As the change required in the CP is for the most of the experiments is order of the noise level, noise amplitude suppresses the desired change in the CP. Another serious problem that may also overshadow minute change in the CP is the parametric drift of the plasma that keep on shifting the threshold point continuously. In case of glow discharge plasma [20–23], where discharge voltage or current acts as a CP, canard induced phenomena have not yet been observed by changing the discharge voltage or current. This may be due to the fact that discharge voltage or current acts as a coarse CP, and hence minute change in the CP may be overshadowed by the presence of the noise and parametric drift in such systems. It is observed that if an excitable glow discharge plasma whose CP (in this case it was discharge voltage) is kept fixed near the threshold, and a magnetic field is applied to it, then the desired change in the CP can be achieved to get canard-induced phenomena. One such canard-induced phenomena is the mixed mode oscillation (MMO) [15–17, 29, 30]. Various mechanisms are responsible for MMOs in deterministic systems like the existence of a Shilnikovtype homoclinic orbit or subcritical Hopf bifurcation [31, 32]. In glow discharge plasmas such kind of canard-induced MMO may also appear near the Homoclinic bifurcations [22] as in this case same conditions can be achieved that are generated by the Shilnikovtype homoclinic orbit or subcritical Hopf bifurcation.

In this paper, we report the appearance of the canard-induced MMOs in a glow discharge plasmas when the

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system was perturbed through a magnetic field at an excitable state keeping discharge voltage fixed. We observed two kinds of oscillations: small quasiperiodic oscillations just after the introduction of the magnetic field, and large bounded limit cycles with increase in the magnetic field. These signals have a well-defined shape that seems similar to the canard-induced mixed mode oscillations (MMOs). We also explored main characteristics, typical frequencies and evolution of inter-oscillations interval to understand the dynamics of the system. These results are validated through the numerical simulation of the FHN system [1, 5, 20]. Phase space plot also shows that these oscillations are MMO.

Rest of the paper has been organized as follows: we have discussed the experimental set up and autonomous plasma dynamics in Section II. The results and discussion of the experiment and simulation results has been presented in Section III. Finally a conclusion has been drawn in Section V.

II. EXPERIMENTAL SETUP AND AUTONOMOUS PLASMA DYNAMICS

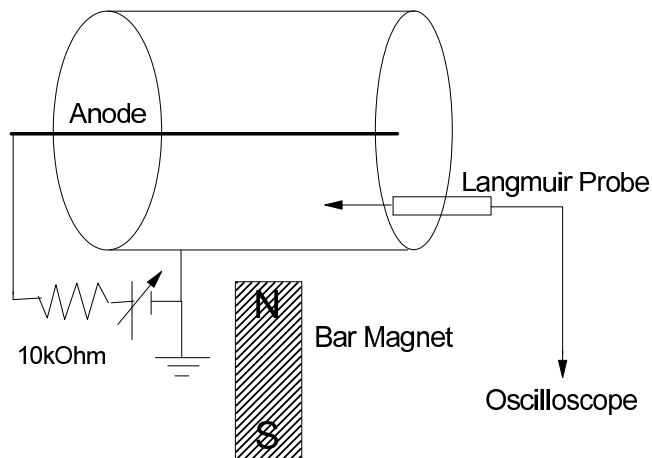


FIG. 1. Schematic diagram of the cylindrical electrode system of the glow discharge plasma. The probe was placed at a distance $l \approx 12.5$ mm from the anode.

The experiments were performed in a hollow cathode dc glow discharge plasma. The schematic diagram of the experimental setup is presented in Fig 1. Here the experimental setup of Ref [22] has been used. Detail of the experimental condition will be found in Ref [22].

The cylindrical hollow electrode [shown in Fig 1] inside which plasma was generated, was kept inside a vacuum chamber and was pumped down to a pressure around 0.001 mbar using a rotary pump. The chamber was subsequently filled with argon gas at $P = 0.36$ mbar. Discharge was initiated by increasing the discharge voltage (DV). At this pressure DV was 401 V to get the excitable dynamics. The system observable was the electrostatic floating potential, which was measured using

a Langmuir probe used in Ref [22]. Time series of the floating potential has been recorded and analyzed to find out underlying dynamics. The plasma density and the electron temperature were determined to be of the order of 10^7 cm^{-3} and 3–4 eV respectively. Furthermore, the electron plasma frequency was observed to be around 28 MHz, whereas the ion plasma frequency was measured to be around 105 kHz. In the present experiment, a magnetic field, which acted as control parameter, was applied to the plasma by using a bar magnet as shown in the same figure.

Through out the experiments, DV and pressure was kept constant. In all the experiments magnetic field was used as the control parameter (CP). In the excitable domain, the system shows irregular and complex oscillations at the initial stages of the DV, and upon increasing the DV, the oscillations became regular period-one oscillation. Further augmentation of the DV modified the oscillation profile and results in the induction of typical relaxation oscillations [22, 33], and these oscillations ceases to steady state to generate excitable state through homoclinic bifurcation that has been discussed in detail in Ref [22]. Once the excitable state is achieved by changing the DV, it kept constant through out the experiment, and a magnetic field is applied to ensure small change of the perturbation to get canard-induced oscillations. From this point the magnetic field acted as CP, and the magnetic field ensures small change in the CP near the threshold to generate canard-enhanced phenomena.

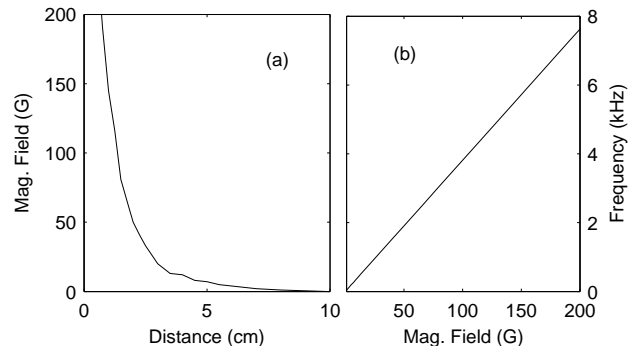


FIG. 2. Variation of the applied Magnetic field from the chamber.

The variable of the external magnetic field, which acts as a CP applied from the outside the chamber [Fig 1]. The variation in the magnetic field with distance is shown in Fig 2 (a). Fig 2 (b) shows the corresponding ion-cyclotron frequency (f_{ci}) of the plasmas that derived using the relation $f_{ci} = 1.52 \times 10^3 Z \mu^{-1} B$ Hz, where, $\mu = \frac{m_i}{m_p}$; Z is charge state and B is the magnetic field in Gauss. m_i is the argon mass and m_p the mass of a proton. Once the excitability is achieved through the change of the DV, it was kept fixed through out the experiment and the magnetic field was varied to get the desired dynamics. It is observed that the magnetic field ensures the small change in the CP near the threshold to generate canard

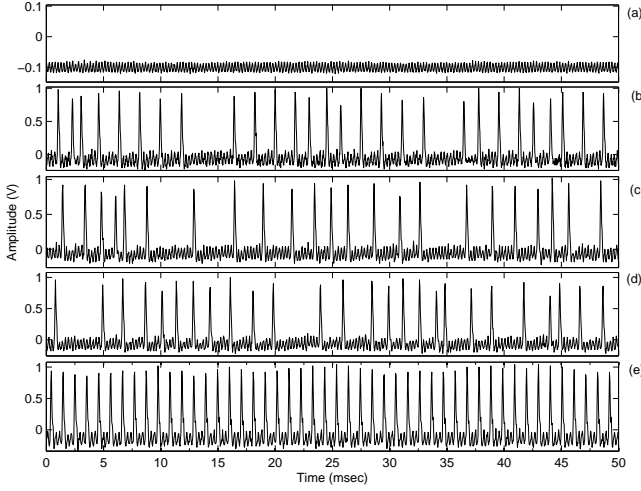


FIG. 3. Plasma oscillations at different magnetic fields. (a) small quasiperiodic oscillations at approximately $B = 2$ Gauss. (b)-(d) Emergence of canard-induced oscillations with the application of magnetic field (4-20) Gauss, and (e) appearance of the regular mixed mode oscillation at $B = 25$ Gauss.

explosion that was not possible by using DV. In the next section, we have presented the experimental results with the applied of magnetic field.

III. RESULT AND DISCUSSION

For the experiments on canard-induced oscillations, the the DV was set to 401 V for the gas pressure $P = 0.36$ mbar so that the output of the plasma floating potential showed excitable fixed point behavior in the absence of magnetic field. The set point was kept a little away from the threshold so that the system remains in a stable state under the influence of parametric drifts and absence of the magnetic field. At this point magnetic field was applied by using a bar-magnet and its intensity was varied through the variation of spatial distance from the anode. Variation of the field with spatial distance has already been shown in Fig 2(a).

The DV was set at $V = 401$ V during the experiment, and at this point plasma showed constant floating potential in the absence of the magnetic field perturbation. Various kind of oscillations depending on the minute change in the magnetic field near the threshold point. Fig 3 shows the plasma floating potential oscillations for different magnetic field. Fig 3(a) shows the quasiperiodic small oscillations at a magnetic field (B) of $\approx 1 - 2$ G (i.e., just after the introduction of the magnetic field). Fig 3(b) shows the same plasma fluctuations at $B = 4$ G. It shows that the large but bounded periodic limit cycle oscillations appears between the small quasiperiodic oscillations. Usually there are 5 small oscillations between two large oscillations. Sometimes, 4 or 3 small quasiperiodic oscillations were also observed. There are

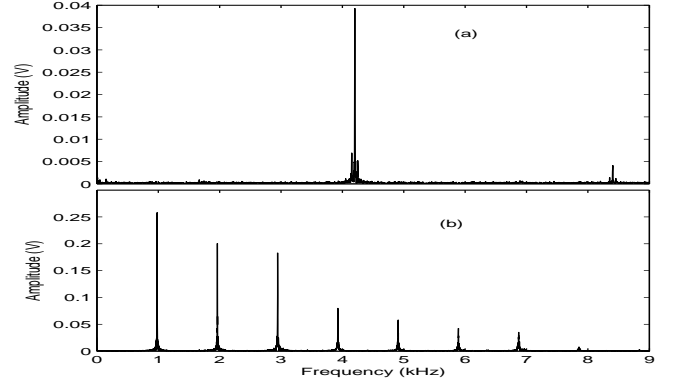


FIG. 4. FFT of experimental data: (a) shows the FFT of quasiperiodic plasma oscillations and (b) oscillations after appearance of canard phenomena. They show the frequency of quasiperiodic plasma oscillations is approximately 4 times higher than canard oscillations.

also long sequence of small quasiperiodic periodic oscillations in between two large sporadic periodic oscillations. Appearance of sporadic long quasiperiodic sequence may be due to parametric drifts of the system from the mean fixed point. These oscillations were observed for wide range of magnetic field (4 G to 20 G) that is shown in Fig 3(b)-(d). When the magnetic field became 25 G, large oscillations were observed after every two small oscillations [Fig 3(e)] and this has been observed till 100 Gauss. As the magnetic field applied from outside the vacuum vessel, it was not possible going beyond the 100 G limit with the present configuration of the experimental setup.

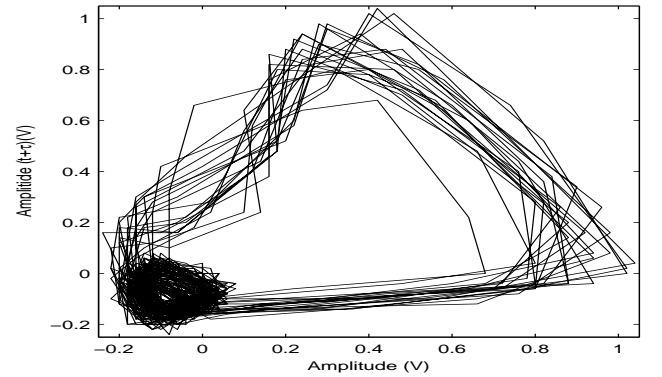


FIG. 5. Phase space plot of canard-induced oscillations obtained from experiment. It shows the emergence of large limit cycles oscillation in between small quasi-periodic oscillations. Smaller oscillations are contaminated by noise. Delay $\tau = 0.04$ ms.

Fig 4 (a) and (b) show respectively the Fourier transform of the large amplitude and quasiperiodic small amplitude plasma potential oscillations. Frequency of the small quasiperiodic oscillations and large amplitude limit cycle oscillations are around 4.2 kHz and 1.0 kHz respec-

tively.

Frequency of the small quasiperiodic oscillations is almost 4 times greater than the large oscillations. Fig 4(a) shows that the frequency of the quasiperiodic small oscillations is almost multiple and 4 times higher than that of the large amplitude limit cycle oscillations and this is consistent with the experimental observation of the canard-induced oscillations [1]. Fig 5 shows the phase space plot of the experimental data, and it shows the generation of the canard-enhanced trajectory. Moreover, the phase space trajectory clearly shows the canard structure present in the system. As there are small quasiperiodic oscillation followed by a number of large amplitude limit cycle oscillation, these phenomena are termed as mixed mode oscillation (MMO).

The main feature of a system which show MMO is that it must be nonlinear with multiple timescale [25, 26, 29]. Occurrence of the multiscale dynamics in an excitable plasma system is already been confirmed [22], where multiscale dynamics has already been exploited to demonstrate noise-induced coherence and stochastic resonances [22, 23]. As in the case of noise-induced phenomena, the same system was observed to undergo homoclinic bifurcation [22], it may be concluded that the origin of the MMO is canard-enhanced phenomena which were generated due to homoclinic bifurcation after the application of the magnetic field perturbation.

In the next section numerical simulation of FHN model is presented to validate our experimental results.

IV. NUMERICAL RESULTS: MODEL ANALYSIS

To validate our experimental results, numerically simulation has been carried out of the FitzHugh Nagumo (FHN) model. Earlier same model has also been used to validate the noise-induced resonances in case of glow discharge plasma experiments [20], where the following FHN model [5, 20] has been studied for an excitable system, whose equations of motion are

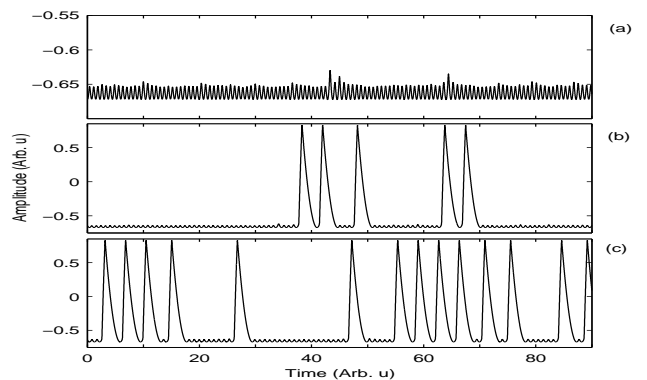


FIG. 6. Oscillation in FHN model. (a) small quasiperiodic oscillations at $a1 = 0.99880$ (b) Emergence of few canard-induced oscillations (large amplitude) $a2 = 0.99950$ and (c) $a3 = 0.99999$.

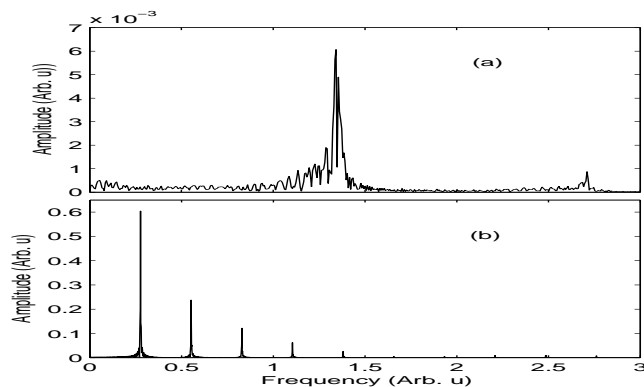


FIG. 7. FFT of the simulated data: (a) shows the FFT of quasiperiodic plasma oscillations and (b) oscillations after appearance of canard phenomena. They show the frequency of quasiperiodic plasma oscillations is also approximately 4 times higher than small oscillations. y -axis of the Figs (c) is scaled down by 100.

$$\begin{aligned} \frac{dx}{dt} &= \frac{1}{\epsilon} \left(x - \frac{x^3}{3} - y \right) \\ \frac{dy}{dt} &= x + a \end{aligned} \quad (1)$$

where, $\epsilon = 0.01$ and the control parameter (' a ') governs the dynamics. here a is the control parameter, and it plays the same role of the magnetic field in the present experiment. For $|a| > 1$ and $|a| < 1$, the system shows fixed point and limit cycle oscillations respectively. Therefore, $a_{th} = 1$ is the threshold and the system shows fixed point and oscillatory behavior above and below this point respectively. Frequency of the limit cycle oscillations increases between $0 \leq a < 1$ and then decreases between $0 \geq a > -1$. Detail behavior with respect to an excitable plasma system has been discussed in Ref [20].

When $a = 0.99880$ the system shows quasiperiodic small oscillations as shown in Fig 6(a), and the frequency of the oscillations is shown in Fig 7(a). When a is increased by a small amount [$a = 0.99880$], the system start showing rapid but bounded growth in the limit cycle oscillation as shown in Fig 6(b) and such growth is termed as canard explosion. No. of limit cycles oscillations increases with increase in the CP value that is typical for canard oscillations [5]. Phase space plot also shows clear the appearance of canard structures. As the amplitude of the model solutions are large compared to the quasiperiodic small oscillations, expanded version is shown in lower panel [Fig 6 (b)]. From the time series and phase space plot it is clear that the system show

MMO.

V. CONCLUSION

Effect of magnetic field near the threshold of an excitable plasma system has been studied. Dynamics of

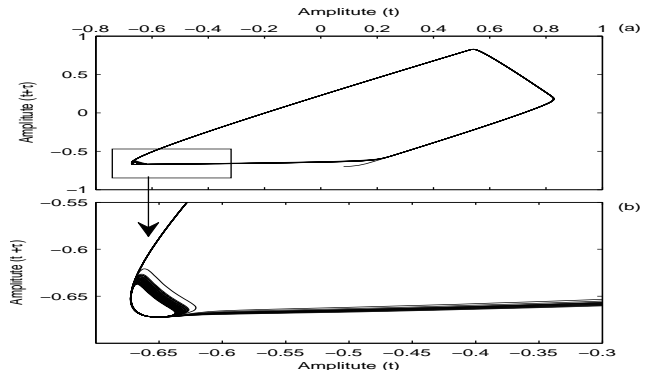


FIG. 8. Phase space plot of canard-induced oscillations obtained from simulation data. It shows the emergence of larger limit cycles from small quasi-periodic oscillations. Time delay $\tau = 0.3$.

the above system is multiscale in nature, and this has already been exploited to get noise-induced resonances using same system [20, 22]. When the same system is perturbed by a magnetic field, it shows canard-induced mixed mode oscillations. Once again the multiscale dynamical behavior has been exploited successfully by applying magnetic field. The result has also been validated through numerical simulation of the FHN model.

Beyond the interest of the study of these nonlinear phenomena from experimental and dynamic point of view, their characterization is also very important for experiments involving real application in glow discharge plasma. Such study may be useful for various application of discharge plasma. The theoretical analysis of such dynamics from actual plasma dynamics may be the subject of future works.

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